PRECALCULUS
Unit 6 Review

NAME KEY
PERIOD

REVIEW Day 2

TRUE OR FALSE: Determine if the following statements are true or false.

1. $\cot\theta = \tan(\theta - 90°)$

FALSE

2. $1-\cos x = \sin x$

FALSE

3. $\tan(25° - 15°) = \dfrac{\tan 25° + \tan 15°}{1 - \tan 25° \tan 15°}$

FALSE

SIMPLIFY:

4. $\cos x - \cos^3 x$

$\cos x(1-\cos^2 x)$

$\cos x(\sin^2 x)$

$\underline{\cos x \sin^2 x}$

5. $\dfrac{\sin^2\alpha + \tan^2\alpha + \cos^2\alpha}{\sec\alpha}$

$\dfrac{\sin^2\alpha + \cos^2\alpha + \tan^2\alpha}{\sec\alpha}$

$\dfrac{1+\tan^2\alpha}{\sec\alpha}$

$\dfrac{\sec^{\cancel{2}}\alpha}{\cancel{\sec\alpha}} = \underline{\sec\alpha}$

6. $1 - 2\sin^2\dfrac{\pi}{3} =$

$\cos 2\left(\dfrac{\pi}{3}\right) = \cos\dfrac{2\pi}{3} = \underline{-\dfrac{1}{2}}$

7. $2\sin 75° \cos 75° =$

$\sin 2(75°) = \sin 150° = \underline{\dfrac{1}{2}}$

8. $\cos^2 25° - \sin^2 25° = \cos 2(25°)$

$= \underline{\cos 50°}$

9. $\dfrac{2\tan\pi}{1-\tan^2\pi} = \tan 2\pi = \underline{0}$

Prove the following identities:

10. $\dfrac{\cos^2 x - 1}{\cos x} = -\tan x \sin x$

$\dfrac{\cancel{1} - \sin^2 x - \cancel{1}}{\cos x} = -\tan x \sin x$

$\dfrac{-\sin^2 x}{\cos x} = -\tan x \sin x$

$-\dfrac{\sin x}{\cos x} \cdot \dfrac{\sin x}{1} = -\tan x \sin x$

$-\tan x \sin x = -\tan x \sin x$ ✓

11. $\dfrac{1}{1-\cos x} + \dfrac{1}{1+\cos x} = 2\csc^2 x$

$\dfrac{1+\cos x}{(1-\cos x)(1+\cos x)} + \dfrac{1-\cos x}{(1-\cos x)(1+\cos x)} = 2\csc^2 x$

$\dfrac{1+\cancel{\cos x}+1-\cancel{\cos x}}{1+\cancel{\cos x}-\cancel{\cos x}-\cos^2 x} = 2\csc^2 x$

$\dfrac{2}{1-\cos^2 x} = 2\csc^2 x$

$\dfrac{2}{\sin^2 x} = 2\csc^2 x$

$2\csc^2 x = 2\csc^2 x$ ✓

12. $\cos^4 x - \sin^4 x = \cos^2 x - \sin^2 x$

$(\cos^2 x + \sin^2 x)(\cos^2 x - \sin^2 x) = \cos^2 x - \sin^2 x$

$\left(\frac{1+\cos 2x}{2} + \frac{1-\cos 2x}{2}\right)(\cos^2 x - \sin^2 x) = \cos^2 x - \sin^2 x$

$\left(\frac{1+\cos 2x + 1 - \cos 2x}{2}\right)(\cos^2 x - \sin^2 x) = \cos^2 x - \sin^2 x$

$\left(\frac{2}{2}\right)(\cos^2 x - \sin^2 x) = \cos^2 x - \sin^2 x$

$\cos^2 x - \sin^2 x = \cos^2 x - \sin^2 x$ ✓

13. $\dfrac{\sin x - \cos x}{\sin x + \cos x} = \dfrac{2\sin^2 x - 1}{1 + 2\sin x \cos x}$

$\dfrac{(\sin x + \cos x)(\sin x - \cos x)}{(\sin x + \cos x)(\sin x + \cos x)} = \dfrac{2\sin^2 x - 1}{1 + 2\sin x \cos x}$

$\dfrac{\sin^2 x + \cancel{\cos x \sin x} - \cancel{\cos x \sin x} - \cos^2 x}{\sin^2 x + \sin x \cos x + \sin x \cos x + \cos^2 x} = \dfrac{2\sin^2 x - 1}{1 + 2\sin x \cos x}$

$\dfrac{\sin^2 x - \cos^2 x}{\sin^2 x + \cos^2 x + 2\sin x \cos x} = \dfrac{2\sin^2 x - 1}{1 + 2\sin x \cos x}$

$\dfrac{\sin^2 x - \cos^2 x}{1 + 2\sin x \cos x} = \dfrac{2\sin^2 x - 1}{1 + 2\sin x \cos x}$

$\dfrac{\sin^2 x - (1 - \sin^2 x)}{1 + 2\sin x \cos x} = \dfrac{2\sin^2 x - 1}{1 + 2\sin x \cos x}$

$\dfrac{\sin^2 x - 1 + \sin^2 x}{1 + 2\sin x \cos x} = \dfrac{2\sin^2 x - 1}{1 + 2\sin x \cos x}$

$\dfrac{2\sin^2 x - 1}{1 + 2\sin x \cos x} = \dfrac{2\sin^2 x - 1}{1 + 2\sin x \cos x}$ ✓

Find all solutions in the interval $[0, 2\pi)$.

14. $\sin x \cos x = \sin x$

$\sin x \cos x - \sin x = 0$

$\sin x(\cos x - 1) = 0$

$\sin x = 0 \qquad \cos x - 1 = 0$

$x = 0, x = \pi \qquad \cos x = 1$

$x = 0$

$\boxed{x = \{0, \pi\}}$

15. $1 - 2\cos x + \cos^2 x = 0$

$(1 - \cos x)^2 = 0$

$1 - \cos x = 0$

$1 = \cos x$

$\boxed{x = 0}$

16. $\sin 2x - \cos x = 0$

$2\sin x \cos x - \cos x = 0$

$\cos x(2\sin x - 1) = 0$

$\cos x = 0 \qquad 2\sin x - 1 = 0$

$x = \pi/2, x = 3\pi/2 \qquad \sin x = \frac{1}{2} \quad x = \pi/6, x = 5\pi/6$

$\boxed{x = \{\pi/6, \pi/2, 5\pi/6, 3\pi/2\}}$

Use the sum and difference formula's to find the exact value of the following.

17. $\sin\left(\underset{\frac{3\pi}{12}+\frac{2\pi}{12}}{\frac{5\pi}{12}}\right) = \sin\left(\frac{\pi}{4} + \frac{\pi}{6}\right) = \sin\frac{\pi}{4}\cos\frac{\pi}{6} + \cos\frac{\pi}{4}\sin\frac{\pi}{6} = \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2}\cdot\frac{1}{2} = \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4}$

$= \dfrac{\sqrt{6} + \sqrt{2}}{4}$

Use the half-angle formulas to find the exact value of the following.

18. $\cos 15^\circ = \cos\left(\frac{30^\circ}{2}\right) = \sqrt{\frac{1 + \cos 30^\circ}{2}} = \sqrt{\frac{1 + \sqrt{3}/2}{2}} = \sqrt{\frac{1}{2} + \frac{\sqrt{3}}{4}} = \sqrt{\frac{1}{4}(2 + \sqrt{3})} = \frac{1}{2}\sqrt{2 + \sqrt{3}}$