5.1 Fundamental Identities

What does the word identity mean to you?

Identity: #45

equality that is true for all values of the domain for both expressions as long as they are both defined

$$\tan\theta \cdot \cos\theta = \sin\theta$$

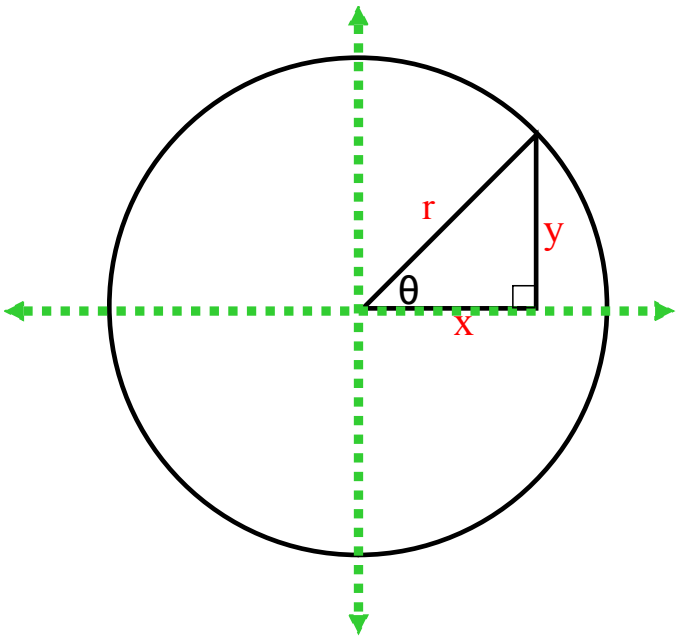
this is true for all θ, as long as sinθ, cosθ, and tanθ are defined

Reciprocal & Quotient Relationships

$$\sin\theta = \frac{1}{\csc\theta} \qquad \csc\theta = \frac{1}{\sin\theta} \qquad \tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\cos\theta = \frac{1}{\sec\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

$$\tan\theta = \frac{1}{\cot\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$



Can we write an equation about the relationship of x, y, and ~~z~~ r?

$x^2 + y^2 = r^2$

Pythagorean Relationships

$$x^2 + y^2 = r^2$$

$\cos^2\theta + \sin^2\theta = 1$

$$\sin^2\theta + \cos^2\theta = 1$$

$-\cos^2\theta \quad -\cos^2\theta$

$$\sin^2\theta = 1 - \cos^2\theta$$

$$\cos^2\theta = 1 - \sin^2\theta$$

$$\frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$$

$\star \tan^2\theta + 1 = \sec^2\theta$

$$\frac{\sin^2\theta}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$$

$\star \quad 1 + \cot^2\theta = \csc^2\theta$

Pythagorean Relationships

$$1+\tan^2\theta=\sec^2\theta$$

$$1=\sec^2\theta-\tan^2\theta$$

$$\tan^2\theta=\sec^2\theta-1$$

$$\sin^2\theta+\cos^2\theta=1$$

$$1+\cot^2\theta=\csc^2\theta$$

$$1=\csc^2\theta-\cot^2\theta$$

$$\cot^2\theta=\csc^2\theta-1$$

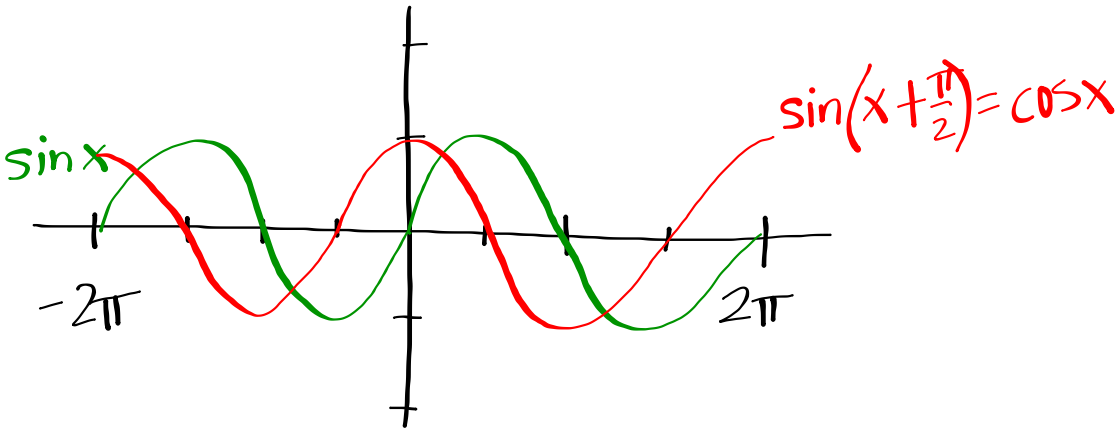
Pythagorean Relationships

$$\sin^2\theta + \cos^2\theta = 1$$

$$1 + \tan^2\theta = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

$$y = \sin\left(x + \frac{\pi}{2}\right)$$



Co-Function Identities

$$\sin(\frac{\pi}{2}-\theta)=\cos\theta \qquad \cos(\frac{\pi}{2}-\theta)=\sin\theta$$

$\sin(-\theta+\frac{\pi}{2})$

$\sin(-(\theta-\frac{\pi}{2}))$

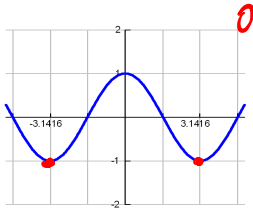
$$\tan(\frac{\pi}{2}-\theta)=\cot\theta \qquad \cot(\frac{\pi}{2}-\theta)=\tan\theta$$

$$\sec(\frac{\pi}{2}-\theta)=\csc\theta \qquad \csc(\frac{\pi}{2}-\theta)=\sec\theta$$

Odd/Even Identities

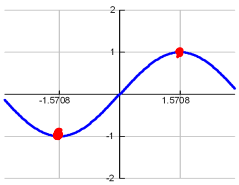
#222

origin y-axis



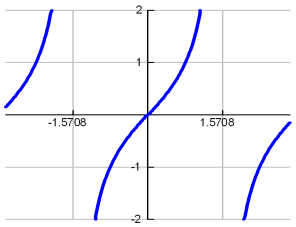
$$\cos(-x)=\cos x$$

$$\sec(-x)=\sec x$$



$$\sin(-x)=-\sin x$$

$$\csc(-x)=-\csc x$$



$$\tan(-x)=-\tan x$$

$$\cot(-x)=-\cot x$$

Simplify:

$\sin x \csc(-x)$ $= \sin x(-\csc x) = \frac{\sin x}{1}\left(\frac{-1}{\sin x}\right) = -1$

$\cot x \tan x = \frac{1}{\tan x}(\tan x) = 1$

$\frac{\cos x}{\sin x} \cdot \frac{\sin x}{\cos x} = 1$

Perfect Squares:

$x^2 - 8x + 16$

$(x-4)(x-4)$

$(x-4)^2$

$x^2 + 14x + 49$

$\sin^2 x - 10\sin x + 25$

let $u = \sin x$

$u^2 - 10u + 25$

$(u-5)(u-5) = (\sin x - 5)(\sin x - 5) = (\sin x - 5)^2$

$\cos^2 x + 16\cos x + 64$

let $u = \cos x$

$u^2 + 16u + 64$

$(u+8)(u+8) = (\cos x + 8)^2$

Difference of Squares:

$x^2 - 16$ $(x-4)(x+4)$

$x^2 - 49$

$1 - x^2$

$1 - \sin^2 x$ $(1+\sin x)(1-\sin x)$

$\sin^2 x - \cos^2 x$ $(\sin x - \cos x)(\sin x + \cos x)$

Simplify:

$$\frac{1-\cos^2 x}{1+\cos x} = \frac{(1-\cos x)(1+\cos x)}{(1+\cos x)} = 1-\cos x$$

$$\frac{(1+\sin x)}{(1+\sin x)}\frac{1}{(1-\sin x)} + \frac{1}{1+\sin x}\frac{(1-\sin x)}{(1-\sin x)}$$

$$\frac{1+\sin x}{1-\sin^2 x} + \frac{1-\sin x}{1-\sin^2 x}$$

$$\frac{1+\sin x + 1 - \sin x}{1-\sin^2 x} = \frac{2}{1-\sin^2 x}$$

$$= \frac{2}{\cos^2 x} = 2\sec^2 x$$

Solve the Equation for $[0, 2\pi)$

$$\tan x \sin^2 x = \tan x$$

$-\tan x \qquad -\tan x$

$\tan x \sin^2 x - \tan x = 0$

$\tan x (\sin^2 x - 1) = 0$

$\tan^{-1}\tan x = 0 \qquad \sin^2 x - 1 = 0$

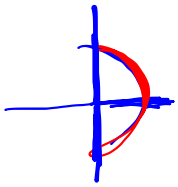
$x = \tan^{-1} 0 \qquad \sqrt{\sin^2 x} = \sqrt{1}$

$\sin x = \pm 1$

$x = 0$

$\sin x = 1 \qquad \sin x = -1$

$x = \sin^{-1} 1 \qquad x = \sin^{-1} -1$

$x = \frac{\pi}{2} \qquad x = -\frac{\pi}{2}$

tan. is undef.