Secondary III Quarter 3 Summative Review Name: KEY

Class:_____

Unit 7

Find the following information and graph each rational function:

Describe how the graph of *g(x)* is related to the graph $f(x)=\frac{1}{x}$. (Transformations)

1. $g(x)=\frac{5}{x}-3$

v. stretch by 5

↓3

2. $g(x)=\frac{-1}{x}+5$

reflect over x-axis

↑5

3. $g(x)=-\frac{1}{(x-2)}+4$

reflect over x-axis

→2

↑4

Given the following functions, find all holes, asymptotes, and intercepts.

4. $f(x)=\frac{x-3}{x^2+6x+5}=\frac{x-3}{(x+5)(x+1)}$

Holes: none

VA: $x=-5, x=-1$

x-int: $(3,0)$

y-int $\left(0,\frac{-3}{5}\right)$

HEB: $y=0$ (HA)

5. $f(x)=-\frac{(x^2-4)}{(x+1)}=\frac{-(x+2)(x-2)}{(x+1)}$

Holes: none

VA: $x=-1$

x-int: $(2,0)$ & $(-2,0)$

y-int $(0,4)$

HEB: slant/oblique (HA)

6. $f(x)=\frac{3x+5}{x+2}$

x	y
-3	4
-4	$\frac{7}{2}$

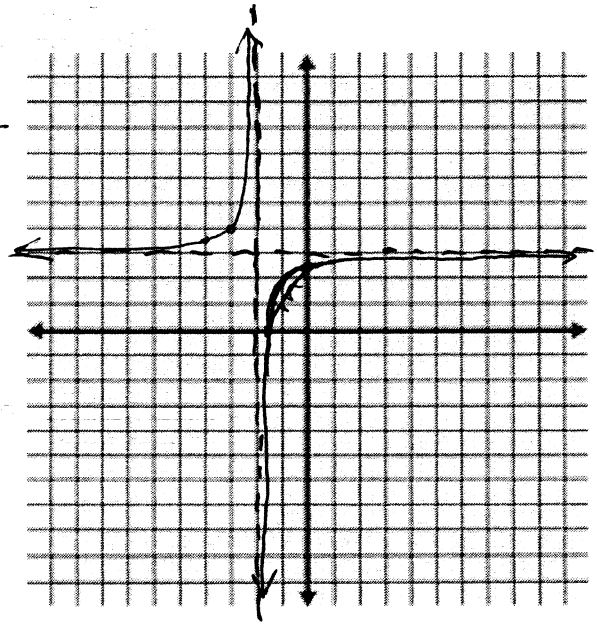
Holes: none

VA: $x=-2$

x-int: $\left(-\frac{5}{3},0\right)$

y-int $\left(0,\frac{5}{2}\right)$

HEB: $y=3$ (HA)



Domain $(-\infty,-2)\cup(-2,\infty)$

Range: $(-\infty,3)\cup(3,\infty)$

Increasing: $(-\infty,-2)\cup(-2,\infty)$

Decreasing —

7. $f(x)=\dfrac{x+3}{x^2-4x-5}$ $(x-5)(x+1)$

x	y
-2	1/7
-4	

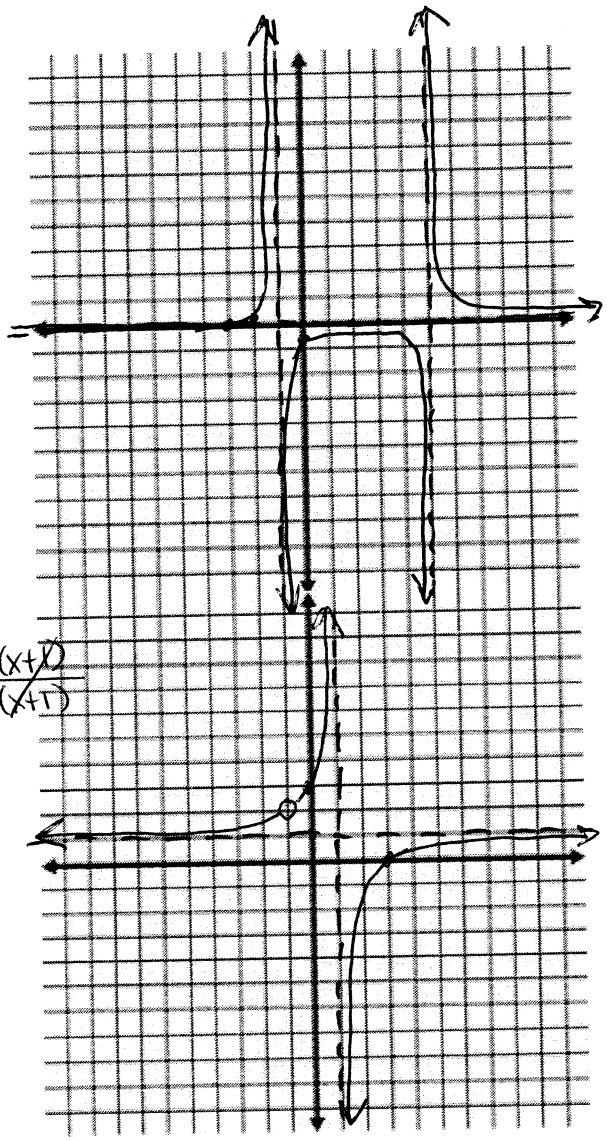
Holes: none

VA: $x=5, x=-1$

x-int: $(-3,0)$

y-int $(0,-\frac{3}{5})$

HEB: $y=0$ (HA)



Domain $(-\infty,-1)\cup(-1,5)\cup(5,\infty)$

Range: $(-\infty,0)\cup(0,\infty)$

Increasing: $(-\infty,-1)\cup(-1,2)$

Decreasing $(2,5)\cup(5,\infty)$

8. $f(x)=\dfrac{x^2-2x-3}{x^2-1}$ $\dfrac{(x-3)(x+1)}{(x-1)(x+1)}$

Holes: $x=-1$

VA: $x=1$

x-int: $(3,0)$

y-int $(0,3)$

HEB: $y=1$ (HA)

Domain $(-\infty,1)\cup(1,\infty)$

Range: $(-\infty,1)\cup(1,\infty)$

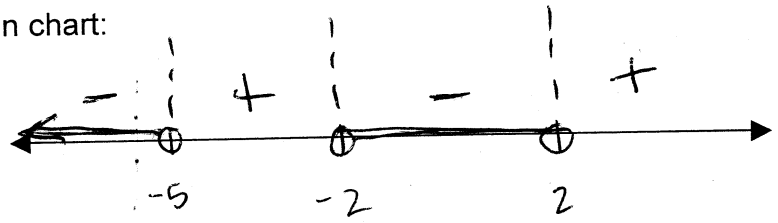
Increasing: $(-\infty,1)\cup(1,\infty)$

Decreasing ——

Solve the following inequalities using a sign chart:

9. $\dfrac{x+5}{x^2-4} \leq 0$ $(x+2)(x-2)$

x	y
0	5/4 neg.



No Holes

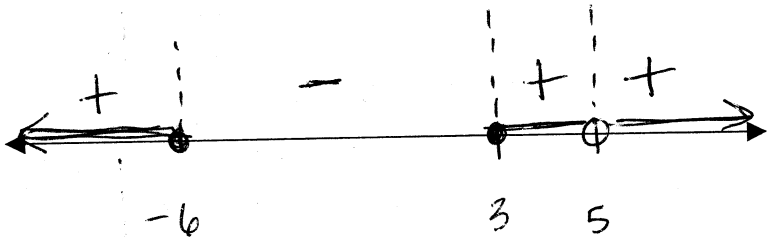
VA: $x=2, x=-2$

x-int.: $(-5,0)$

$(-\infty,-5)\cup(-2,2)$

10. $\dfrac{x^2+3x-18}{x^2-10x+25} \geq 0$ $\dfrac{(x+6)(x-3)}{(x-5)(x-5)}$

x	y
0	-18/25 neg.



No Holes

VA: $x=5$

x-int.: $(-6,0)$ & $(3,0)$

Unit 8

Write an explicit and recursive rule for the following

1. 9, 27, 81, 243,…

Explicit: $f(n)=9(3)^n$

Recursive: $f(n)=f(n-1)\cdot 3$

2. 4, –3, –10, –17, ...

Explicit: $f(n)=4-7n$

Recursive: $f(n)=f(n-1)-7$

5. Find the stated term for the following sequence –3, –6, –12, –24, ...; 9th term $-48, -96, -192, -384,$

-768

9. A geometric sequence that has an first term 2, ends with -4374 and has a common ratio of -3, how many terms are in the sequence? $f(n)=2(-3)^n$

$\frac{-4374}{2}=\frac{2(-3)^n}{2}$

$-2187=(-3)^n$

$n=7$

Evaluate the following

7. $\sum_{n=1}^{5} 2n+1$

$3+5+7+9+11=35$

8. $\sum_{k=1}^{3} k^2-1$ $0+3+8=11$

3. Which exponential equation DOES NOT have a vertical stretch? (Multiple choice)

a. $y=13(2)^x$ b. $y=\frac{1}{2}(2)^x$ c. $y=2\left(\frac{1}{2}\right)^x$ (d.) $y=\left(\frac{4}{2}\right)^x$

10. For the function $f(x)=5^x$, what is the limit as $x\to\infty$?

(A.) $\lim_{x\to\infty} f(x)=\infty$ B. $\lim_{x\to\infty} f(x)=-\infty$ C. $\lim_{x\to\infty} f(x)=0$ D. $\lim_{x\to\infty} f(x)=1$

Find the domain, range, and y-intercept for the following functions. Also state whether the function represents growth or decay.

11. $f(x)=2(3)^{x-2}-1$

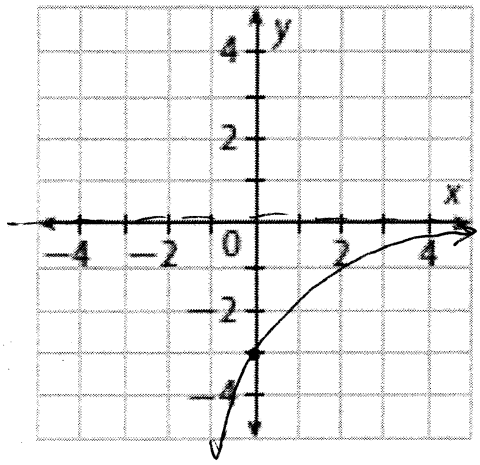
Domain: $(-\infty,\infty)$
Range: $(-1,\infty)$
y-int: $(0,-.78)$
Asymptote: $y=-1$
Growth or Decay? (circle one)

12. $f(x)=\left(\frac{1}{3}\right)^{x}+2$

Domain: $(-\infty,\infty)$
Range: $(2,\infty)$
y-int: $(0,3)$
Asymptote: $y=2$
Growth or Decay? (circle one)

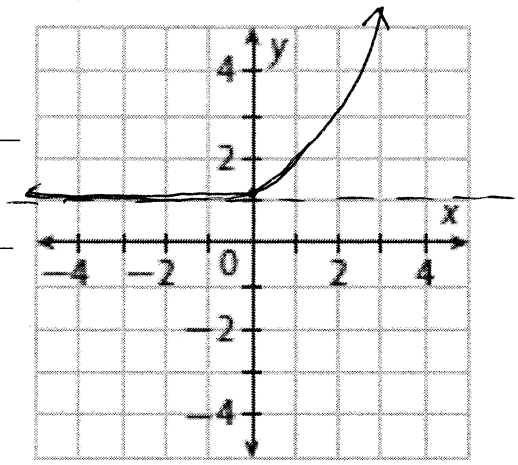
Graph the following and label any asymptotes or intercepts

13. $g(x)=-3\left(\frac{1}{2}\right)^{x}$



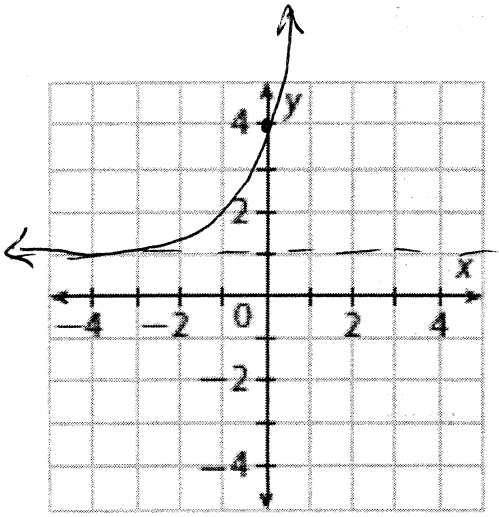
y-int: $(0,-3)$
HA $y=0$

14. $f(x)=2^{x-2}+1$



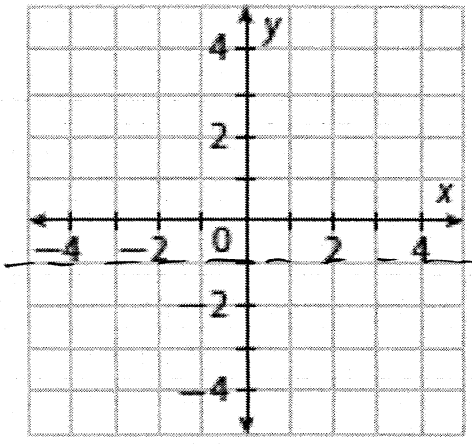
y int $(0,\frac{5}{4})$
HA $y=1$

15. $f(x)=3(4)^{x}+1$



y-int: $(0,4)$
HA: $y=1$

16. $f(x)=-1\left(\frac{1}{3}\right)^{x+3}-1$



y-int.: $(0,-\frac{10}{9})$
HA: $y=-1$

15. If Jane invests $4,200 at an 8% interest **compounded continuously**, how much money will there be after 10 years?

$A = 4200e^{.08(10)}$

$A = \$9347.27$

(16-18) Answer the following questions with the following: an investment of $2000 that earns 3.4% interest

16. Write an equation to describe the value V(t) of the investment at time t if the interest is **compounded monthly**. $V(t) = 2000\left(1 + \frac{.034}{12}\right)^{12t}$

17. What is the value of the investment after 9 years?

$V(t) = 2000\left(1 + \frac{.034}{12}\right)^{12(9)} \approx \$2714.79$

18. How long would it take for the investment to reach $11,000?

$\frac{11000}{2000} = \frac{2000\left(1 + \frac{.034}{12}\right)^{12t}}{2000}$

$5.5 = (1.003)^{12t}$

$\log_{1.003} 5.5 = \frac{\log(1.003)}{\log 1.003}^{12t}$

$\frac{569.101}{12} = \frac{12t}{12}$

$47.4 \text{ yrs} = t$

19. A melting snowman is losing one-half of his weight each day. He originally weighed 256 pounds. Assuming that the outside temperature stays the same, how much does the snowman weigh after 5 days?

$A = 256\left(\frac{1}{2}\right)^5 = 8 \text{ lbs}$

20. A car with a cost of $25,000 is decreasing in value at a rate of 10% each year. When will the value of the car be about $13,000?

$\frac{13000}{25000} = \frac{25000(1 - .10)^t}{25000}$

$.52 = .9^t$

$\log_{.9} .52 = \frac{\log .9^t}{\log .9}$

$t = 6.2 \text{ yrs}$

21. An online video game tournament begins with 4096 players. Four players play in each game. In each game there is only one winner, and only the winner advances to the next round. How many games will the winner play?

$A = 4096\left(\frac{1}{4}\right)^t$

$\frac{1}{4096} = \frac{4096\left(\frac{1}{4}\right)^t}{4096}$

$\frac{1}{4096} = \left(\frac{1}{4}\right)^t$

$t = 6 \text{ games}$

Unit 9

Write the given exponential equation as a logarithmic equation

1. $4^2 = 16$ $\log_4 16 = 2$

2. $e^{17} = a$ $\ln a = 17$

3. $10^4 = 10{,}000$ $\log 10000 = 4$

4. $b^p = a$ $\log_b a = p$

Write the given logarithmic equation as an exponential equation

5. $\log_7 x = 10$ $7^{10} = x$

6. $\ln x = 32$ $e^{32} = x$

7. $\log 1000 = 3$ $10^3 = 1000$

8. $\log_2 5 = x$ $2^x = 5$

9. Evaluate without using a calculator: if $f(x) = \log_5 x$, find $f(125)$, $f\left(\frac{1}{25}\right)$, $f(\sqrt{5})$

$f(125) = \log_5 125 = 3$

$f\left(\frac{1}{25}\right) = \log_5 \frac{1}{25} = -2$

$f(\sqrt{5}) = \log_5 \sqrt{5} = \log_5 5^{1/2} = \frac{1}{2}$

10. Evaluate without using a calculator: if $f(x) = \log_3 x$ find $f(27), f(3), f(\sqrt{3})$

$f(27) = \log_3 27 = 3$

$f(3) = \log_3 3 = 1$

$f(\sqrt{3}) = \log_3 \sqrt{3} = \log_3 3^{1/2} = \frac{1}{2}$

Evaluate the following without a calculator:

11. $\log_4 1$ 0

12. $\ln e$ 1

13. $\log_5 5$ 1

14. $7^{\log_7 12}$ 12

15. $\log_{12} 12^{15}$ 15

16. $\ln e^{32}$ 32

17. $10^{\log 14}$ 14

17. $\log_5 \sqrt{5}$ $\frac{1}{2}$

Write each as a single logarithm. Assume that all variables are positive.

18. $3\log_4 2+\log_4 6$

$\log_4 2^3+\log_4 6$

$\log_4 2^3\cdot 6=\log_4 8\cdot 6$

$\log_4 48$

19. $\frac{1}{3}\log_7 y-6\log_7 z$

$\log_7 y^{1/3}-\log_7 z^6$

$\log_7 \frac{y^{1/3}}{z^6}$

20. $(3\log_2 x+\frac{1}{2}\log_2 y)-2\log_2(xz)$

$(\log_2 x^3+\log_2 y^{1/2})-\log_2(xz)^2$

$\log_2 x^3y^{1/2}-\log_2(xz)^2$

$\log_2 \frac{x^3y^{1/2}}{(xz)^2}$

Use the properties of logarithms to expand the following. Express all exponents as coefficients.

21. $\log_3 x^2y^4$

$\log_3 x^2+\log_3 y^4$

$2\log_3 x+4\log_3 y$

22. $\log_{12}\frac{\sqrt{x}}{y^2}$

$\log_{12}\sqrt{x}-\log_{12}y^2$

$\log_{12}x^{1/2}-2\log_{12}y$

$\frac{1}{2}\log_{12}x-2\log_{12}y$

23. $\log_4\frac{x\sqrt{y}}{z^{12}w^2}$ $\log_4 x\sqrt{y}-\log_4 z^{12}w^2$

$\log_4 x+\log_4\sqrt{y}-(\log_4 z^{12}+\log_4 w^2)$

$\log_4 x+\frac{1}{2}\log_4 y-12\log_4 z-2\log_4 w$

Use the ~~Change-of-Base to~~ calculator rewrite the following expressions as common logarithms.

24. $\log_5 3$

.68

25. $\log_{12}13$

1.03

26. $\log 80000$

4.9

Solve the following. Round your answer to the nearest hundredth. Check for extraneous solutions.

27. $4^{2x+10}+6=262$

$4^{2x+10}=256$

$\log_4 4^{2x+10}=\log_4 256$

$2x+10=\log_4 256$

$\frac{2x}{2}=\frac{\log_4 256-10}{2}$

$x=\frac{\log_4 256-10}{2}\approx -3$

28. $\frac{7e^{\frac{x}{4}}}{7}=\frac{500}{7}$

$e^{x/4}=71.429$

$\ln e^{x/4}=\ln 71.429$

$4\cdot\frac{x}{4}=\ln 71.429\cdot 4$

$x=4\ln 71.429\approx 17.07$

29. $\log_2(x-2)=4$

$2^4 = x-2$

$x = 2^4 - 2$

$x = 14$

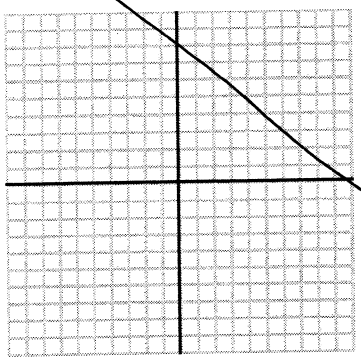
30. $\ln(x-1)=8$

$e^8 = x-1$

$x = e^8 + 1 \approx 2981.96$

Without a calculator, graph the following, list the transformations (if any), asymptote and two points:

31. $f(x)=\log_3 x$

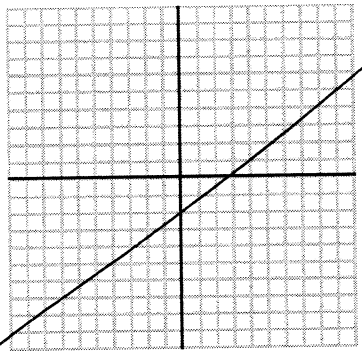


Transformations:

Points:

Asymptote:

32. $f(x)=-\ln x$

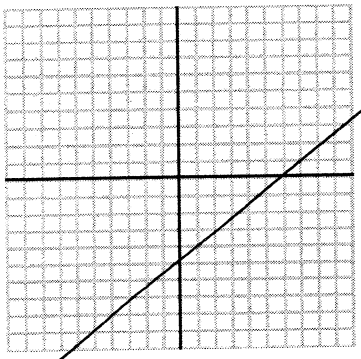


Transformations:

Points:

Asymptote:

33. $f(x)=\log_2(x-3)-2$

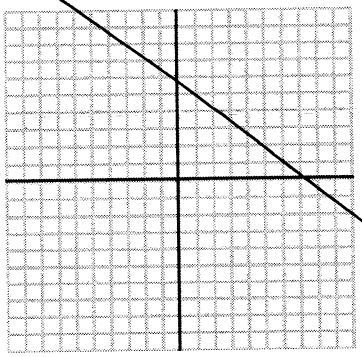


Transformations:

Points:

Asymptote:

34. $f(x)=2\log_3(x+2)+2$



Transformations:

Points:

Asymptote:

35. The pH of orange juice is 3.2, and the pH of milk is 6.1.

$pH=-\log[H^+]$ What are the hydrogen-ion concentrations of orange juice and milk?

$-3.2 = +\log[H^+]$ OJ: $10^{-3.2} = H^+ \approx .0006$

$-6.1 = +\log[H^+]$ Milk $10^{-6.1} = H^+ \approx .0000008$

36. If Bob invests \$5,000 with a 4% interest rate compounded monthly, how long will it take until his investment has grown to \$7,000? $A=P\left(1+\frac{r}{n}\right)^{nt}$

$\frac{7000}{5000} = \frac{5000\left(1+\frac{.04}{12}\right)^{12t}}{5000}$

$1.4 = (1.003)^{12t}$

$\log_{1.003} 1.4 = \log_{1.003}(1.003)^{12t} \rightarrow \frac{12t}{12} = \frac{\log_{1.003} 1.4}{12} \rightarrow t = 9.36 \text{ yrs}$